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Deemed to be University



SANTOSH
Academia

Santosh Academia Solutions to JEE (Main) - 2021

Test Date: 18th March 2021 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper- 1

Time Allotted: 3 Hours

Maximum Marks: 300

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

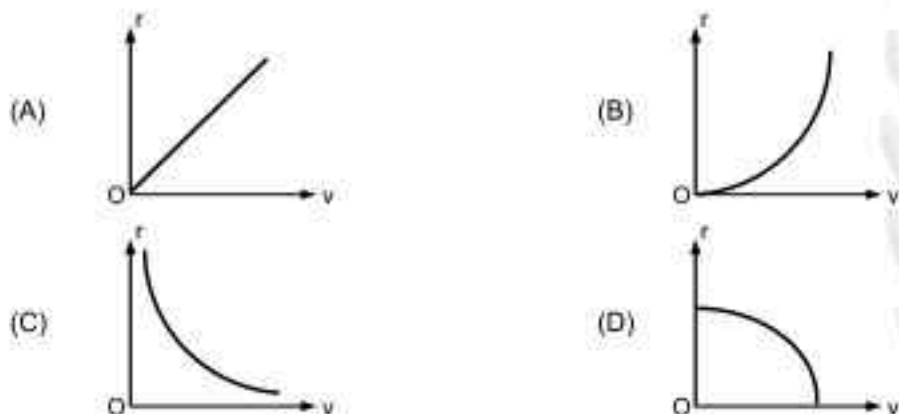
Important Instructions:

- The test is of 3 hours duration.
- This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
- This question paper contains **Three Parts**. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
- Section – A** : Attempt all questions.
- Section – B** : Do any 5 questions out of 10 Questions.
- Section-A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.
- Section-B (01 – 10)** contains 10 Numerical based questions with answer as numerical value. Each question carries **+4 marks** for correct answer. There is no negative marking.

- Q4.** A proton and an α -particle, having kinetic energy K_p and K_α , respectively, enter into a magnetic field at right angles. The ratio of the radii of trajectory of proton to that of α -particle is 2 : 1. The ratio of $K_p : K_\alpha$ is
 (A) 1:8 (B) 8:1
 (C) 1:4 (D) 4:1

- Q5.** The time taken for the magnetic energy to reach 25% of its maximum value, when solenoid of resistance R , inductance L is connected to a battery, is:
 (A) $\frac{L}{R} \ln(2)$ (B) $\frac{L}{R} \ln(10)$
 (C) Infinite (D) $\frac{L}{R} \ln(5)$

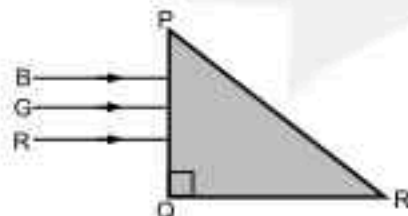
- Q6.** A particle of mass m moves in a circular orbit under the central potential field, $V(r) = -\frac{C}{r}$, where C is a constant. The correct radius – velocity graph of the particle's motion is;



- Q7.** For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (γ is the ratio of specific heats):

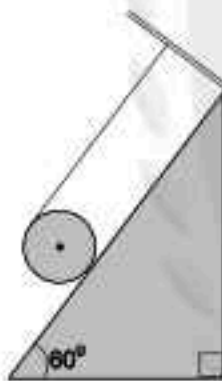
- (A) $-\gamma \frac{dV}{V}$ (B) $\frac{dV}{V}$
 (C) $-\gamma \frac{V}{dV}$ (D) $-\frac{1}{\gamma} \frac{dV}{V}$

- Q8.** Three rays of light, red (R), green (G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in the figure. The refractive indices of material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49 respectively. The colour of ray(s) emerging out of the face PR is



- (A) red (B) blue and green
 (C) green (D) blue

- Q9.** The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is \vec{L} . The magnitude of the areal velocity of the planet is ;
- (A) $\frac{2\vec{L}}{M}$ (B) $\frac{L}{2M}$
 (C) $\frac{4L}{M}$ (D) $\frac{L}{M}$
- Q10.** The decay of a proton to neutron is ;
- (A) possible only inside the nucleus
 (B) not possible but neutron to proton conversion is possible
 (C) not possible as proton mass is less than the neutron mass
 (D) always possible as it is associated only with β^+ decay
- Q11.** The function of time representing a simple harmonic motion with time period of $\frac{\pi}{\omega}$ is :
- (A) $\sin(\omega t) + \cos(\omega t)$ (B) $\sin^2(\omega t)$
 (C) $3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$ (D) $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$
- Q12.** If the angular velocity of earth's spin is increased such that the bodies at the equator start floating , the duration of the day would be approximately :
 [Take $g = 10 \text{ ms}^{-2}$, the radius of earth, $R = 6400 \times 10^3 \text{ m}$, Take $\pi = 3.14$]
- (A) 1200 minutes (B) does not change
 (C) 60 minutes (D) 84 minutes
- Q13.** The correct relation between α (ratio of collector current to emitter current) and β (ratio of collector current to base current) of a transistor is ;
- (A) $\beta = \frac{1}{1-\alpha}$ (B) $\alpha = \frac{\beta}{1+\beta}$
 (C) $\alpha = \frac{\beta}{1-\alpha}$ (D) $\beta = \frac{\alpha}{1+\alpha}$
- Q14.** An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is S_1 and that of the other part is S_2 . Given that $S_1 > S_2$. If the piston is removed then the total entropy of the system will be :
- (A) $S_1 - S_2$ (B) $\frac{S_1}{S_2}$
 (C) $S_1 \times S_2$ (D) $S_1 + S_2$
- Q15.** An object of mass m_1 collides with another object of mass m_2 , which is at rest. After collision the objects move with equal speeds in the opposite direction. The ratio of the masses $m_2 : m_1$ is ;
- (A) 1:1 (B) 3:1
 (C) 1:2 (D) 2:1

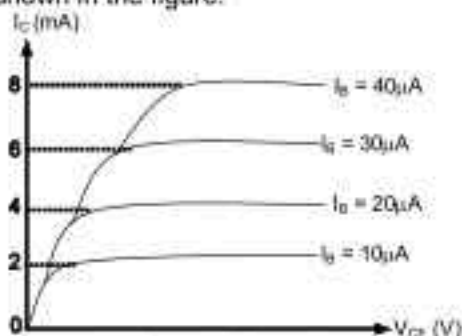
- Q16.** In a series LCR circuit, the inductive reactance (X_L) is $10\ \Omega$, and the capacitive reactance (X_C) is $4\ \Omega$. The resistance (R) in the circuit is $6\ \Omega$. The power factor of the circuit is :
- (A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$
 (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2\sqrt{2}}$
- Q17.** Consider a uniform wire of mass M and length L . It is bent into a semi-circle. Its moment of inertia about a line perpendicular to the plane of the wire passing through centre is :
- (A) $\frac{ML^2}{\pi^2}$ (B) $\frac{2ML^2}{5\pi^2}$
 (C) $\frac{ML^2}{2\pi^2}$ (D) $\frac{ML^2}{4\pi^2}$
- Q18.** A plane electromagnetic wave propagating along y -direction, can have the following pair of electric field (\vec{E}) and magnetic field (\vec{B}) components :
- (A) E_x, B_y or E_y, B_x (B) E_x, B_z or E_z, B_x
 (C) E_x, B_z or E_z, B_x (D) E_x, B_z or E_z, B_x
- Q19.** A solid cylinder of mass m is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is:
 [the coefficient of static friction, μ_s is 0.4]
- (A) 0 (B) $\frac{7mg}{2}$
 (C) $5mg$ (D) $\frac{mg}{5}$
- 
- Q20.** Consider a sample of oxygen behaving like an ideal gas. At 300K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be :
 (Molecular weight of oxygen is $32\ \text{g/mol}$; $R = 8.3\ \text{JK}^{-1}\text{mol}^{-1}$)
- (A) $\sqrt{\frac{8\pi}{3}}$ (B) $\sqrt{\frac{8}{3}}$
 (C) $\sqrt{\frac{3}{8}}$ (D) $\sqrt{\frac{3\pi}{8}}$

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

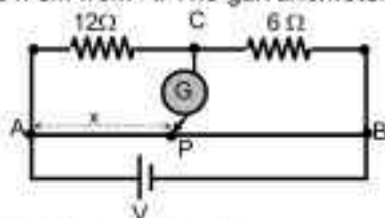
- Q1.** The typical output characteristics curve for a transistor working in the common –emitter configuration is shown in the figure.



The estimated current gain from the figure is.....

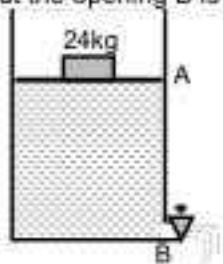
- Q2.** A TV transmission tower antenna is at a height of 20m. Suppose that the receiving antenna is at
 (i) ground level
 (ii) a height of 5m
 The increase in the antenna range in case (ii) relative to case (i) is $n\%$.
 The value of n , to the nearest integer, is.....

- Q3.** Consider a 72 cm long wire AB as shown in the figure. The galvanometer jockey is placed at P on AB at a distance x cm from A. The galvanometer shows zero deflection.



The value of x , to nearest integer, is.....

- Q4.** Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 m^2 . The tank has an opening B near the bottom whose cross-sectional area is 1 cm^2 . A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is $v \text{ ms}^{-1}$.

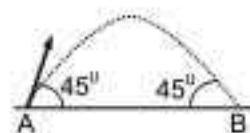


The value of v , to the nearest integer, is.....
 [Take value of g to be 10 ms^{-2}]

Q5. Two wires of same length and thickness having specific resistance $6 \Omega \text{ cm}$ and $3 \Omega \text{ cm}$ respectively are connected in parallel. The effective resistivity is $\rho \Omega \text{ cm}$. The value of ρ , to the nearest integer, is.....

Q6. The radius of a sphere is measured to be $(7.50 \pm 0.85) \text{ cm}$. Suppose the percentage error in its volume is x . The value of x , to the nearest integer, is.....

Q7. The projectile motion of a particle of mass 5 g is shown in the figure. The initial velocity of the projectile is $5\sqrt{2} \text{ ms}^{-1}$ and air resistance is assumed to be negligible. The magnitude of change in momentum between the points A to B is



$x \times 10^{-2} \text{ kg m s}^{-1}$

The value of x , to the nearest integer, is.....

Q8. A galaxy is moving away from the earth at a speed 286 kms^{-1} . the shift in wavelength of a redline at 630 nm is $x \times 10^{-18} \text{ m}$. The value of x , to the nearest integer, is.....

[Take the value of speed of light c , as $3 \times 10^8 \text{ ms}^{-1}$]

Q9. A ball of mass 4 kg , moving with a velocity 10 ms^{-1} , collides with a spring of length 8 m and force constant 100 Nm^{-1} . The length of compressed spring is $x \text{ m}$. The value of x , to the nearest integer, is.....

Q10. An infinite number of point charges, each carrying $1 \mu\text{C}$ charge, are placed along y -axis at $y = 1 \text{ m}, 2 \text{ m}, 4 \text{ m}, 8 \text{ m}, \dots$. The total force on a 1 C point charge, placed at the origin is $x \times 10^3 \text{ N}$. The value of x , to the nearest integer, is.....

$\left[\text{Take } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 / \text{C}^2 \right]$

PART – B (CHEMISTRY)**SECTION - A****(One Options Correct Type)**

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Given below are two statements:

Statement I: Bohr's theory accounts for the stability and line spectrum of Li^+ ion.

Statement II: Bohr's theory was unable to explain the splitting of spectral lines in the presence of a magnetic field.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (A) Both statement I and statement II are true.
 (B) Statement I is true but statement II is false.
 (C) Both statement I and statement II are false
 (D) Statement I is false but statement II is true.

Q2. Match List – I with List – II:

List – I

- (a) Be
 (b) Mg
 (c) Ca
 (d) Ra

List – II

- (i) Treatment of cancer
 (ii) Extraction of metals
 (iii) Incendiary bombs and signals
 (iv) Windows of X-ray tubes
 (v) Bearings for motor engines.

Choose the **most appropriate** answer from the option given below:

- (A) (a)-(iii), (b)-(iv), (c)-(v), (d)-(ii)
 (B) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)
 (C) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(v)
 (D) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Q3. Given below are two statements:

Statement I : C_2H_5OH and $AgCN$ both can generate nucleophile.

Statement II : KCN and $AgCN$ both will generate nitrile nucleophile with all reaction conditions.

Choose the **most appropriate** option:

- (A) Both statement I and statement II are false.
 (B) Statement I is false but statement II is true.
 (C) Both statement I and statement II are true.
 (D) Statement I is true but statement II is false.

Q4. The oxide that shows magnetic property is:

- (A) SiO_2
 (B) Na_2O
 (C) Mn_3O_4
 (D) MgO

Q5. In basic medium H_2O_2 exhibits which of the following reactions?

- (A) $Mn^{2+} \rightarrow Mn^{4+}$
 (B) $I_2 \rightarrow I^-$
 (C) $PbS \rightarrow PbSO_4$

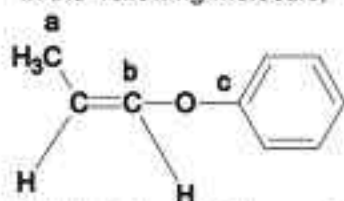
Choose the **most appropriate** answer from the options given below:

- (A) (A), (C) only
 (B) (B) only
 (C) (A), (B) only
 (D) (A) only

- Q6.** A hard substance melts at high temperature and is an insulator in both solid and in molten state. This solid is most likely to be a/ an:
- (A) Ionic solid (B) Metallic solid
(C) Molecular solid (D) Covalent solid

- Q7.** Given below are two statements:
Statement I : Non- biodegradable wastes are generated by the thermal power plants.
Statement II : Bio-degradable detergents leads to eutrophication.
 In the light of the above statements, choose the **most appropriate** answer from the options given below:
- (A) Statement I is true but statement II is false
 (B) Both Statement I and Statement II are false.
 (C) Statement I is false but Statement II is true.
 (D) Both Statement I and Statement II are true.

- Q8.** In the following molecule,



Hybridisation of Carbon a, b and c respectively are:

- (A) sp^3, sp^2, sp^2 (B) sp^3, sp, sp
 (C) sp^3, sp, sp^2 (D) sp^3, sp^2, sp
- Q9.** Match List – I with List – II:
- | List – I
(Class of Chemicals) | List – II
(Example) |
|----------------------------------|------------------------|
| (a) Antifertility drug | (i) Meprobamate |
| (b) Antibiotic | (ii) Alitame |
| (c) Tranquilizer | (iii) Norethindrone |
| (d) Artificial Sweetener | (iv) Salvarsan |
- Choose the **most appropriate** match:
- (A) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i) (B) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
 (C) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i) (D) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

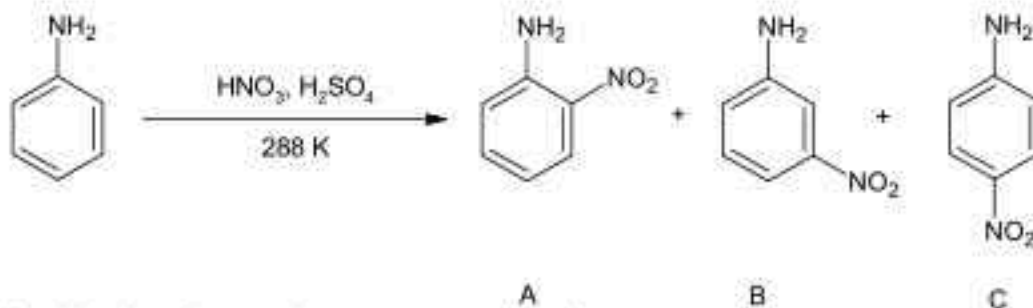
- Q10.** The oxidation states of nitrogen in NO , NO_2 , N_2O and NO_3 are in the order of:
- (A) $NO_2 > NO_3 > NO > N_2O$ (B) $NO_3 > NO_2 > NO > N_2O$
 (C) $NO > NO_2 > N_2O > NO_3$ (D) $N_2O > NO_2 > NO > NO_3$

- Q11.** The secondary valency and the number of hydrogen bonded water molecule(s) in $CuSO_4 \cdot 5H_2O$ respectively, are:
- (A) 6 and 4 (B) 4 and 1
 (C) 5 and 1 (D) 6 and 5

- Q12.** Deficiency of vitamin K causes:
- (A) Cheilosis (B) Increase in blood clotting time
 (C) Decrease in blood clotting time (D) Increase in fragility of RBC's

- Q13.** In the reaction of hypobromite with amide, the carbonyl carbon is lost as:
- (A) CO_3^{2-} (B) HCO_3^-
 (C) CO (D) CO_2

Q14.



Consider the given reaction, percentage yield of:

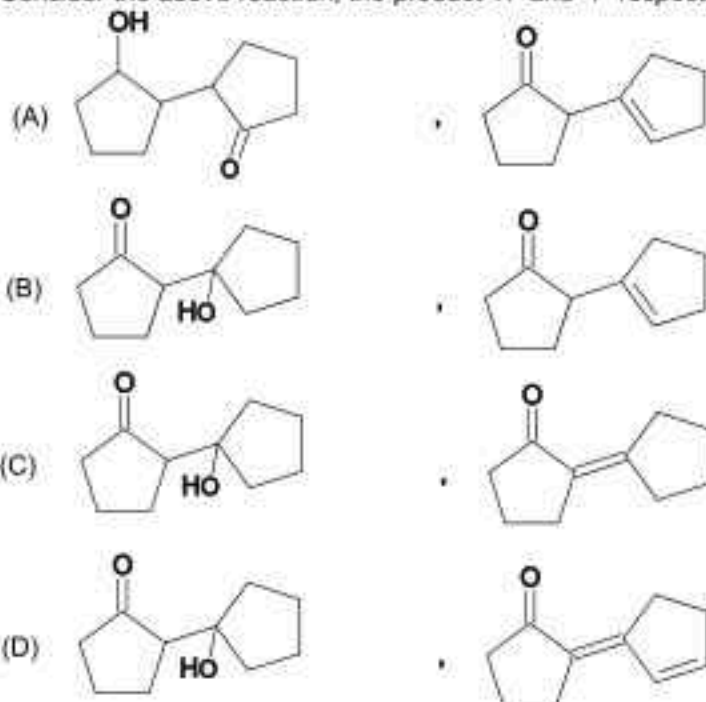
- (A) $B > C > A$
 (C) $A > C > B$

- (B) $C > B > A$
 (D) $C > A > B$

Q15.



Consider the above reaction, the product 'X' and 'Y' respectively are:



Q16. Match List - I with List - II:

List - I

- (a) Mercury
 (b) Copper
 (c) Silicon
 (d) Nickel

List - II

- (i) Vapour phase refining
 (ii) Distillation Refining
 (iii) Electrolytic Refining
 (iv) Zone Refining

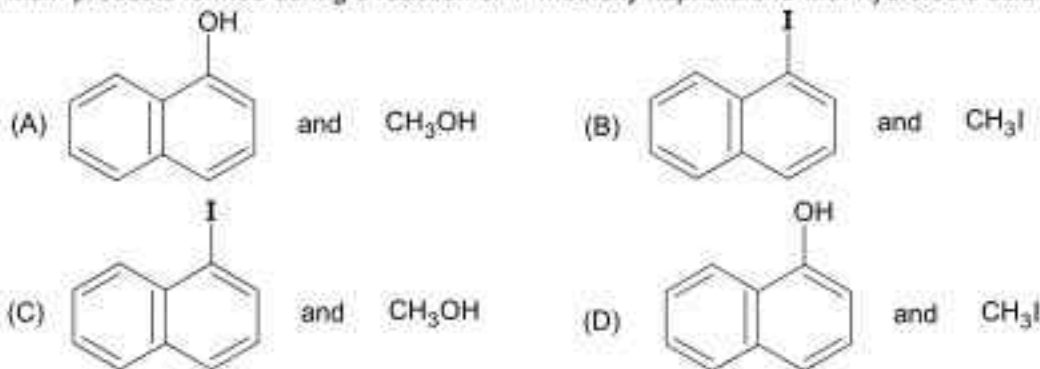
Choose the most appropriate answer from the option given below:

- (A) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)
 (C) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

- (B) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
 (D) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

- Q17.** The first ionization energy of magnesium is smaller as compared to that of elements X and Y, but higher than that of Z. The elements X, Y and Z, respectively are:
 (A) argon, chlorine and sodium (B) chlorine, lithium and sodium
 (C) neon, sodium and chlorine (D) argon, lithium and sodium

- Q18.** Main products formed during a reaction of 1-methoxy naphthalene with hydroiodic acid are:



- Q19.** The charges on the colloidal CdS sol and TiO_2 sol are, respectively:
 (A) Positive and negative (B) Positive and positive
 (C) Negative and positive (D) Negative and negative

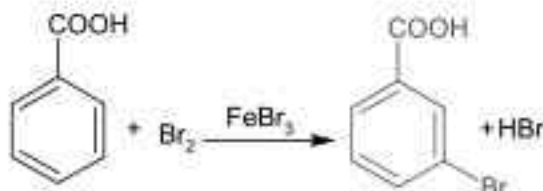
- Q20.** An organic compound "A" on treatment with benzene sulphonyl chloride gives compound B. B is soluble in dil. NaOH solution. Compound A is:



SECTION - B**(Numerical Answer Type)**

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

- Q1.** A solute A dimerizes in water. The boiling point of a 2 molal solution of A is 100.52°C . The percentage association of A is _____. (Round off to the nearest integer).
[Use: K_b for water = $0.52 \text{ K kg mol}^{-1}$, Boiling point of water = 100°C]
- Q2.** The number of species below that have two lone pairs of electrons in their central atom is _____. (Round off to the nearest integer).
 SF_4 , BF_4^- , ClF_3 , AsF_3 , PCl_5 , BrF_3 , XeF_4 , SF_6
- Q3.** A xenon compound 'A' upon partial hydrolysis gives XeO_2F_2 . The number of lone pair of electrons present in compound A is _____. (Round off to the nearest integer).
- Q4.** In Tollen's test for aldehyde, the overall number of electron(s) transferred to the Tollen's reagent formula $[\text{Ag}(\text{NH}_3)_2]^+$ per aldehyde group to form silver mirror is _____. (Round off to the nearest integer).
- Q5.** 10.0 mL of Na_2CO_3 solution is titrated against 0.2 M HCl solution. The following titre values were obtained in 5 readings: 4.8 mL, 4.9 mL, 5.0 mL, 5.0 mL and 5.0 mL. Based on these readings, and convention of titrimetric estimation the concentration of Na_2CO_3 solution is _____ mM. (Round off to the nearest integer).
- Q6.** The molar conductivities at infinite dilution of barium chloride, sulphuric acid and hydrochloric acid are 280, 860 and $426 \text{ S cm}^2 \text{ mol}^{-1}$ respectively. The molar conductivity at infinite dilution of barium sulphate is _____ $\text{S cm}^2 \text{ mol}^{-1}$. (Round off to the nearest integer).
- Q7.** A reaction has a half life of 1 min. The time required for 99.9% completion of the reaction is _____ min. (Round off to the nearest integer).
[Use $\ln 2 = 0.69$; $\ln 10 = 2.3$]

Q8.

Consider the above reaction where 6.1 g of Benzoic acid is used to get 7.8g of m- bromo benzoic acid. The percentage yield of the product is _____.

[Given : Atomic masses : C : 12.0u, H:1.0u, O:16.0u, Br:80.0 u]

- Q9.** The gas phase reaction
 $2A(g) \rightleftharpoons A_2(g)$
At 400 K has $\Delta G^\circ = +25.2 \text{ kJ mol}^{-1}$.
The equilibrium constant K_C for this reaction is _____ $\times 10^{-2}$.
(Round off to the nearest integer).
[Use : $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$, $\ln 10 = 2.3$, $\log_{10} 2 = 0.30$, $1 \text{ atm} = 1 \text{ bar}$]
[antilog $(-0.3) = 0.501$]
- Q10.** The solubility of CdSO_4 in water is $8.0 \times 10^{-4} \text{ mol L}^{-1}$. Its solubility in $0.01 \text{ M H}_2\text{SO}_4$ solution is _____ $\times 10^{-6} \text{ mol L}^{-1}$.
(Round off to the nearest integer).
(Assume that solubility is much less than 0.01 M)

PART – C (MATHEMATICS)**SECTION - A****(One Options Correct Type)**

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

- Q1.** Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC , then $(R + r)$ is equal to :
- (A) $7\sqrt{2}$ (B) $2\sqrt{2}$
 (C) $\frac{9}{\sqrt{2}}$ (D) $3\sqrt{2}$
- Q2.** A pole stands vertically inside a triangular park ABC. Let the angel of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of ΔABC is 2, then the height of the pole is equal to :
- (A) $\frac{1}{\sqrt{3}}$ (B) $\sqrt{3}$
 (C) $2\sqrt{3}$ (D) $\frac{2\sqrt{3}}{3}$
- Q3.** Define a relation R over a class of $n \times n$ real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$ ". Then which of the following is true?
- (A) R is reflexive, symmetric but not transitive
 (B) R is symmetric, transitive but no reflexive
 (C) R is reflexive, transitive but not symmetric
 (D) R is an equivalence relation
- Q4.** Consider a hyperbola $H: x^2 - 2y^2 = 4$. Let the tangent at a point $P(4, \sqrt{6})$ meet the x-axis at Q and latus rectum at $R(x_1, y_1), x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to :
- (A) $4\sqrt{6} - 1$ (B) $4\sqrt{6}$
 (C) $\sqrt{6} - 1$ (D) $\frac{7}{\sqrt{6}} - 2$
- Q5.** Let $f: \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{1\}$ be defined by $f(x) = \frac{x-2}{x-3}$.
 Let $g: \mathbb{R} \rightarrow \mathbb{R}$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to :
- (A) 2 (B) 7
 (C) 5 (D) 3

- Q6.** Let in a series of $2n$ observations, half of them are equal to a and remaining half are equal to $-a$. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to :
- (A) 650 (B) 925
(C) 425 (D) 250
- Q7.** Let \vec{a} and \vec{b} are two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))$ and \vec{a} is equal to :
- (A) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (B) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$
(C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (D) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$
- Q8.** If $15 \sin^4 \alpha + 10 \cos^4 \alpha = 6$, for some $\alpha \in \mathbb{R}$, then the value of $27 \sec^6 \alpha + 8 \operatorname{cosec}^4 \alpha$ is equal to :
- (A) 500 (B) 350
(C) 400 (D) 250
- Q9.** If P and Q are two statements, then which of the following compound statement is a tautology?
- (A) $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow Q$ (B) $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow (P \wedge Q)$
(C) $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow P$ (D) $((P \Rightarrow Q) \wedge \neg Q) \Rightarrow \neg P$
- Q10.** Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = (y+1)\left((y+1)e^{x/2} - x\right)$, $0 < x < 2.1$, with $y(2) = 0$. Then the value of $\frac{dy}{dx}$ at $x = 1$ is equal to :
- (A) $\frac{-e^{3/2}}{(e^2 + 1)^2}$ (B) $\frac{2e^2}{(1 + e^2)^2}$
(C) $\frac{e^{5/2}}{(1 + e^2)^2}$ (D) $\frac{5e^{1/2}}{(e^2 + 1)^2}$
- Q11.** Let S_1 be the sum of first $2n$ terms of an arithmetic progression. Let S_2 be the sum of first $4n$ terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first $6n$ terms of the arithmetic progression is equal to :
- (A) 5000 (B) 1000
(C) 7000 (D) 3000
- Q12.** Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as
- $$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin 2x}{2x} & , \text{ if } x < 0 \\ \frac{b}{x} & , \text{ if } x = 0 \\ \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}} & , \text{ if } x > 0 \end{cases}$$
- If f is continuous at $x = 0$, then the value of $a + b$ is equal to :
- (A) -2 (B) -3
(C) $-\frac{5}{2}$ (D) $-\frac{3}{2}$

- Q13.** Let a tangent be drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ where $\theta \in \left(0, \frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to :
- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$
 (C) $\frac{\pi}{8}$ (D) $\frac{\pi}{3}$
- Q14.** Let the system of linear equations
 $4x + \lambda y + 2z = 0$
 $2x - y + z = 0$
 $\mu x + 2y + 3z = 0, \lambda, \mu \in \mathbb{R}$
 has a non-trivial solution. Then which of the following is true?
 (A) $\mu = 6, \lambda \in \mathbb{R}$ (B) $\mu = -6, \lambda \in \mathbb{R}$
 (C) $\lambda = 2, \mu \in \mathbb{R}$ (D) $\lambda = 3, \mu \in \mathbb{R}$
- Q15.** Let $g(x) = \int_0^x f(t) dt$, where f is continuous function in $[0, 3]$ such that $\frac{1}{3} \leq f(t) \leq 1$ for all $t \in [0, 1]$ and $0 \leq f(t) \leq \frac{1}{2}$ for all $t \in (1, 3]$. The largest possible interval in which $g(3)$ lies is :
- (A) $[1, 3]$ (B) $\left[\frac{1}{3}, 2\right]$
 (C) $\left[-1, -\frac{1}{2}\right]$ (D) $\left[-\frac{3}{2}, -1\right]$
- Q16.** In a triangle ABC, if $|BC| = 8, |CA| = 7, |AB| = 10$, then the projection of the vector \vec{AB} on \vec{AC} is equal to :
- (A) $\frac{85}{14}$ (B) $\frac{25}{4}$
 (C) $\frac{127}{20}$ (D) $\frac{115}{16}$
- Q17.** Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :
- (A) 4 (B) $\frac{1}{4}$
 (C) $\frac{1}{2}$ (D) 2
- Q18.** The area bounded by the curve $4y^2 = x^2(4-x)(x-2)$ is equal to :
- (A) $\frac{\pi}{16}$ (B) $\frac{\pi}{8}$
 (C) $\frac{3\pi}{2}$ (D) $\frac{3\pi}{8}$

Q19. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(A) $\frac{40}{243}$

(B) $\frac{80}{243}$

(C) $\frac{32}{625}$

(D) $\frac{128}{625}$

Q20. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x - 2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points :

(A) $\left(\frac{1}{2}, \pm \frac{\sqrt{5}}{2}\right)$

(B) $(0, \pm\sqrt{3})$

(C) $(1, \pm 2)$

(D) $\left(2, \pm \frac{3}{2}\right)$

SECTION - B

(Numerical Answer Type)

This section contains 10 questions. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

- Q1.** Let nC_k denote the binomial coefficient of x^k in the expansion of $(1+x)^n$. If $\sum_{k=0}^{10} (2^2 + 3k) {}^nC_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$, $\alpha, \beta \in \mathbb{R}$, then $\alpha + \beta$ is equal to.....
- Q2.** Let I be an identity matrix of order 2×2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in \mathbb{N}$ for which $P^n = 5I - 8P$ is equal to.....
- Q3.** Let $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfy the equation $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$ and $f(x) \neq 0$ for any $x \in \mathbb{R}$. If the function f is differentiable at $x = 0$ and $f'(0) = 3$, then $\lim_{h \rightarrow 0} \frac{1}{h} (f(h) - 1)$ is equal to.....
- Q4.** Let the mirror image of the point $(1, 3, a)$ with respect to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) - b = 0$ be $(-3, 5, 2)$. Then, the value of $|a + b|$ is equal to.....
- Q5.** Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P , then the value of $|5\alpha|$ is equal to....
- Q6.** Let $P(x)$ be a real polynomial of degree 3 which vanishes at $x = -3$. Let $P(x)$ have local minima at $x = 1$, local maxima at $x = -1$ and $\int_{-1}^1 P(x) dx = 18$, then the sum of all the coefficients of the polynomial $P(x)$ is equal to.....
- Q7.** The term independent of x in the expansion of $\left[\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x-x^{\frac{1}{2}}} \right]^{10}$, $x \neq 1$, is equal to.....
- Q8.** If $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$, then the value of α is equal to.....
- Q9.** If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^2)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to.....
- Q10.** Let $y = y(x)$ be the solution of the differential equation $x dy - y dx = \sqrt{x^2 - y^2} dx$, $x \geq 1$, with $y(1) = 0$. If the area bounded by the line $x = 1$, $x = e^t$, $y = 0$ and $y = y(x)$ is $\alpha e^{2t} + \beta$, then the value of $10(\alpha + \beta)$ is equal to.....

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Keys to JEE (Main) - 2021

PART – A (PHYSICS)

SECTION - A

1. A	2. A	3. A	4. D
5. A	6. C	7. A	8. A
9. B	10. A	11. B, C	12. D
13. B	14. D	15. B	16. C
17. A	18. D	19. D	20. D

SECTION - B

1. 200	2. 50	3. 48	4. 3
5. 4	6. 34	7. 5	8. 6
9. 2	10. 12		

PART – B (CHEMISTRY)

SECTION - A

1. D	2. D	3. D	4. C
5. C	6. D	7. D	8. A
9. B	10. B	11. B	12. B
13. A	14. B	15. C	16. A
17. A	18. D	19. C	20. B

SECTION - B

1. 100	2. 2	3. 1	4. 2
5. 50	6. 288	7. 10	8. 78
9. 166	10. 64		

PART – C (MATHEMATICS)

SECTION - A

1.	C	2.	C	3.	D	4.	D
5.	C	6.	C	7.	C	8.	D
9.	D	10.	A	11.	D	12.	D
13.	A	14.	A	15.	B	16.	A
17.	C	18.	C	19.	C	20.	D

SECTION – B

1.	19	2.	6	3.	3	4.	1
5.	38	6.	8	7.	210	8.	160
9.	0	10.	4				

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Solutions to JEE (Main) - 2021

PART – A (PHYSICS)

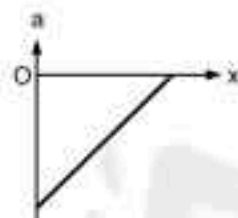
SECTION - A

1.

Equation of curve is

$$\frac{v}{v_0} + \frac{x}{x_0} = 1 \Rightarrow v = -\left(\frac{v_0}{x_0}\right)x + v_0$$

$$\Rightarrow a = \frac{dv}{dt} = -\left(\frac{v_0}{x_0}\right)(v) = -\left(\frac{v_0}{x_0}\right)\left(-\left(\frac{v_0}{x_0}\right)x + v_0\right) \Rightarrow a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

**Concept involved:** Graph of kinematics**Topic:** Kinematics**Difficulty level:** Moderate**Note:** IIT-Jee-2005**Point of Error:** Writing Equation of straight line and differentiation

2.

Basic Fact

Concept involved: Electric and magnetic field lines**Topic:** Magnetics and Magnetic Material**Difficulty level:** Moderate**Point of Error:** Fact

3.

As we know that resolving power (R) of a microscope is given as

$$R = \frac{2\mu \sin \theta}{1.22\lambda} \dots\dots\dots (1)$$

According to de-Broglie's hypothesis, we can write

$$\lambda = \frac{h}{mv} \dots\dots\dots (2)$$

With the help of equations (1) and (2), we can write

$$R = \frac{2\mu mv \sin \theta}{1.22h} \Rightarrow \frac{R_r}{R_v} = \frac{m_r}{m_e} = 1837 \Rightarrow R_r = 1837R_v$$

Concept involved: Resolving Power of Microscope**Topic:** Optics**Difficulty level:** Moderate**Point of Error:** Formula

4.

As we know that radius of trajectory is given as

$$R = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB} \Rightarrow K = \frac{R^2 q^2 B^2}{2m}$$

$$\Rightarrow \frac{K_p}{K_o} = \frac{R_p^2 q_p^2 B^2}{2m_p} \times \frac{2m_o}{R_o^2 q_o^2 B^2} = \left(\frac{m_o}{m_p}\right) \times \left(\frac{R_p}{R_o}\right)^2 \times \left(\frac{q_p}{q_o}\right)^2$$

$$\Rightarrow \frac{K_p}{K_o} = (4) \times (2)^2 \times \left(\frac{1}{2}\right)^2 = 4:1$$

Concept involved: Motion of charged particle in magnetic Field

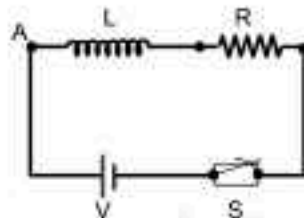
Topic: Magnetism

Difficulty level: Easy

5. As we know that the current in L-R - circuit is given as

$$I = \frac{V}{R} \left(1 - e^{-\frac{t}{\tau}}\right), \text{ where } \tau = \frac{L}{R}$$

$$E = \frac{LI^2}{2} = \frac{LV^2}{2R^2} \left(1 - e^{-\frac{t}{\tau}}\right)^2 \Rightarrow \text{Energy stored in inductor}$$



According to Question, we can write

$$\frac{1}{4} \left(\frac{LV^2}{2R^2}\right) = \frac{LV^2}{2R^2} \left(1 - e^{-\frac{t}{\tau}}\right)^2 \Rightarrow \frac{1}{4} = \left(1 - e^{-\frac{t}{\tau}}\right)^2 \Rightarrow \frac{1}{2} = 1 - e^{-\frac{t}{\tau}} \Rightarrow e^{-\frac{t}{\tau}} = \frac{1}{2}$$

$$\Rightarrow \frac{t}{\tau} = \ln(2) \Rightarrow t = \tau \ln(2) = \frac{L}{R} \ln(2)$$

Concept involved: Energy stored in inductor in L-R - circuit

Topic: EMI

Difficulty level: Moderate

Point of Error: Formula

6.
$$U = mV(r) = -\frac{Cm}{r}$$

$$F = -\frac{dU}{dr} = \frac{Cm}{r^2} \Rightarrow \text{The force which provides required centripetal force}$$

$$\Rightarrow \frac{mv^2}{r} = \frac{Cm}{r^2} \Rightarrow r = \frac{C}{v^2}$$

Concept involved: Dynamics of circular motion

Topic: Work ,power and Energy

Difficulty level: Moderate

7. As we know that for adiabatic expansion

$$PV^\gamma = C \Rightarrow V^\gamma \frac{dP}{dV} + P(\gamma V^{\gamma-1}) = 0 \Rightarrow \frac{dP}{dV} = -\gamma \frac{P}{V} \Rightarrow \frac{dP}{P} = -\gamma \frac{dV}{V}$$

Concept involved: Adiabatic Process

Topic: Heat and Thermodynamics

Difficulty level: Easy

8. For light ray to come out from face PR of prism

$$\theta_c > i = \theta \Rightarrow \sin \theta_c > \sin \theta \Rightarrow \frac{1}{\mu} > \sin \theta$$

$$\Rightarrow \mu < \frac{1}{\sin \theta}$$

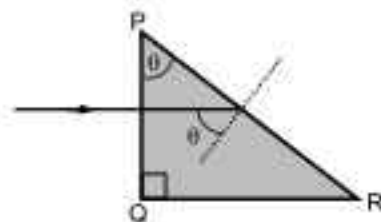
Note: If we assume $\theta = 45^\circ \Rightarrow \mu < 1.414$, then red colour light ray will come out from face PR of prism

Concept involved: Total internal refraction

Topic: Optics

Difficulty level: Moderate

Note: IIT-Jee-1989



9. $A = \text{Area swept} \Rightarrow \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} \left(\frac{Mr^2\omega}{M} \right) = \frac{L}{2M}$

Concept involved: Kepler's Second Law

Topic: Gravitation

Difficulty level: Easy

10. Basic Fact

Concept involved: β^+ decay

Topic: Modern Physics

Difficulty level: Easy

11. (A) $\sin(\omega t) + \cos(\omega t) = \sqrt{2} \sin\left(\omega t + \frac{\pi}{4}\right) \Rightarrow T = \frac{2\pi}{\omega}$

(B) $\sin^2(\omega t) = \frac{1}{2} - \frac{1}{2} \cos(2\omega t) \Rightarrow T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

(C) $3 \cos\left(\frac{\pi}{4} - 2\omega t\right) \Rightarrow T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

(D) $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$

$$\text{Time period of } \cos(\omega t) = \frac{2\pi}{\omega}$$

$$\text{Time period of } \cos(2\omega t) = \frac{2\pi}{2\omega}$$

$$\text{Time period of } \cos(3\omega t) = \frac{2\pi}{3\omega}$$

$$\text{Time period of combined function} = \frac{2\pi}{\omega}$$

Concept involved: Basic equation of SHM

Topic: SHM

Difficulty level: Easy

12. $g_c = g - \omega^2 R \Rightarrow 0 = g - \omega^2 R \Rightarrow \omega = \sqrt{\frac{g}{R}}$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{R}{g}} = 2 \times 3.14 \times \sqrt{\frac{6400 \times 10^1}{10}}$$

$$\Rightarrow T = 2 \times 3.14 \times 80 \times 10^5 = 83.733 \text{ min} = 84 \text{ min}$$

Concept involved: Variation of g [Circular motion of earth]

Topic: Laws of motion

Difficulty level: Moderate

13. As we know that for a transistor

$$I_e = I_c + I_b \Rightarrow \frac{I_e}{I_c} = 1 + \frac{I_b}{I_c} \Rightarrow \frac{1}{\alpha} = 1 + \frac{1}{\beta} = \frac{1+\beta}{\beta} \Rightarrow \alpha = \frac{\beta}{1+\beta}$$

$$\Rightarrow \frac{1}{\beta} = \frac{1}{\alpha} - 1 = \frac{1-\alpha}{\alpha} \Rightarrow \beta = \frac{\alpha}{1-\alpha}$$

Concept involved: Transistor

Topic: Semiconductor and devices

Difficulty level: Easy

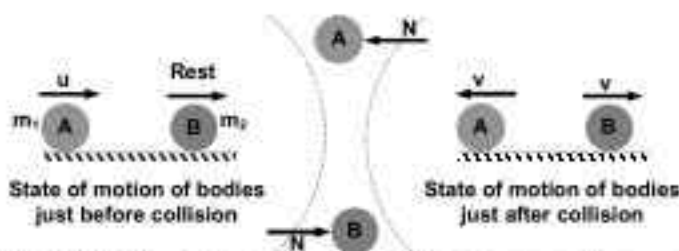
14. $S = S_1 + S_2$

Concept involved: Entropy

Topic: Heat and thermodynamics

Difficulty level: Moderate

- 15.



With the help of Conservation of linear momentum, we can write

$$m_1 u = (m_2 - m_1) v \dots\dots\dots (1)$$

With the help of definition of e , we can write

$$e = \frac{v_2}{v_1} = \frac{2v}{u} \Rightarrow u = \frac{2v}{e} \dots\dots\dots (2)$$

Putting the value of e in equation (1), we have

$$m_1 \frac{2v}{e} = (m_2 - m_1) v \Rightarrow 2m_1 = em_2 - em_1 \Rightarrow \frac{m_2}{m_1} = \frac{2+e}{e} = 1 + \frac{2}{e} > 2$$

From options, only possible answer is B

Concept involved: Collision

Topic: Centre of mass and collision

Difficulty level: Difficult

16. $z = \sqrt{R^2 + (X_c - X_l)^2} = \sqrt{(6)^2 + (4 - 10)^2} = 6\sqrt{2} \Omega$

$$\text{Power factor} = \cos \phi = \frac{R}{z} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Concept involved: Power Factor of LCR-circuit

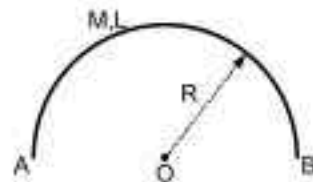
Topic: Alternating Current

Difficulty level: Easy

17.

$$\pi R = L \Rightarrow R = \frac{L}{\pi}$$

$$I_O = MR^2 = M \left(\frac{L}{\pi} \right)^2 = \frac{ML^2}{\pi^2}$$



Concept involved: Moment of Inertia
Topic: Rotation
Difficulty level: Moderate

18. As we know that direction of propagation of electromagnetic wave is perpendicular to plane containing mutually perpendicular electric field and magnetic field, so option D will be correct answer.

Concept involved: Propagation of EMW
Topic: EMW
Difficulty level: Easy

19. As we know that for rolling without slipping on inclined plane, the minimum value of coefficient of static friction will be

$$\mu_{\min} = \frac{I \tan \theta}{I + mR^2}$$

$$\Rightarrow \mu_{\min} = \frac{\frac{mR^2}{2} \tan \theta}{\frac{mR^2}{2} + mR^2} = \frac{\tan \theta}{3} = \frac{\tan 60^\circ}{3} = \frac{\sqrt{3}}{3} = \frac{1.732}{3} = 0.5773$$

Since given coefficient of static friction is less than μ_{\min} , so body will perform rolling with slipping and kinetic friction will act

$$F_k = \mu N = \mu mg \cos \theta = (0.4) \times mg \cos 60^\circ = \frac{mg}{5}$$

Concept involved: Rolling motion on inclined plane
Topic: Rotation
Difficulty level: Difficult

20. As we know that

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \text{ and } v_{\text{av}} = \sqrt{\frac{8RT}{\pi M}}$$

$$\Rightarrow \frac{v_{\text{rms}}}{v_{\text{av}}} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{8RT}{\pi M}}} = \sqrt{\frac{3\pi}{8}}$$

Concept involved: Kinetic theory of gases
Topic: Heat and thermodynamics
Difficulty level: Easy

SECTION - B

1.
$$\beta = \frac{I_C}{I_B} = \frac{2 \times 10^{-7}}{10 \times 10^{-9}} = 200$$

2. d_{in} = Distance covered by transmitting antenna + Distance covered by receiving antenna

$$\Rightarrow d_{in} = \sqrt{2R_e h_{transmitting}} + \sqrt{2R_e h_{receiving}}$$

$$\Rightarrow d_{in} = \sqrt{2 \times 64 \times 10^5 \times 20} + \sqrt{2 \times 64 \times 10^5 \times 5} = 16000 + 8000 = 24000m$$

When $h_{receiving} = 0$ then

$$d_{2m} = \sqrt{2 \times 64 \times 10^5 \times 20} = 16000m$$

$$\% \text{ increment} = \frac{8000}{16000} \times 100 = 50$$

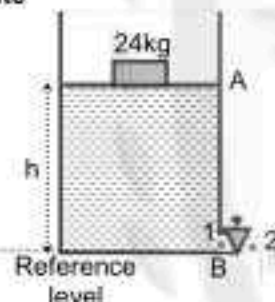
3.
$$\frac{12}{x} = \frac{6}{72-x} \Rightarrow x = 144 - 2x \Rightarrow x = \frac{144}{3} = 48cm$$

4. Using Bernoulli's equation at point-1 and point-2, we can write

$$\frac{v_1^2}{2} + \frac{P_1}{\rho} + gh_1 = \frac{v_2^2}{2} + \frac{P_2}{\rho} + gh_2$$

$$\Rightarrow \frac{v_1^2}{2} + \frac{P_0 + \rho gh + \frac{mg}{A}}{\rho} + g \times 0 = \frac{v_2^2}{2} + \frac{P_0}{\rho} + g \times 0$$

$$\Rightarrow \frac{v_1^2}{2} + \frac{\rho gh + \frac{mg}{A}}{\rho} = \frac{v_2^2}{2} \dots\dots\dots(1)$$



Using Continuity equation , we can write

$$Av_1 = av_2 \Rightarrow v_1 = \frac{a}{A} v_2$$

Putting the value of v_1 in equation(1),we have

$$\Rightarrow \frac{a^2 v_2^2}{A^2 \cdot 2} + \frac{\rho gh + \frac{mg}{A}}{\rho} = \frac{v_2^2}{2} \Rightarrow \left(\frac{A^2 - a^2}{A^2} \right) v_2^2 = \frac{2 \left(\rho gh + \frac{mg}{A} \right)}{\rho}$$

$$\Rightarrow v_2 = \sqrt{\left(\frac{A^2}{A^2 - a^2} \right) \left\{ \frac{2 \left(\rho gh + \frac{mg}{A} \right)}{\rho} \right\}} = 3.09 m/s \approx 3 m/s$$

$$5. \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \Rightarrow \frac{\rho(\ell)}{2A} = \frac{\left\{ \frac{\rho_1 \ell}{A} \right\} \left\{ \frac{\rho_2 \ell}{A} \right\}}{\left\{ \frac{\rho_1 \ell}{A} \right\} + \left\{ \frac{\rho_2 \ell}{A} \right\}} \Rightarrow \rho = \frac{2\rho_1 \rho_2}{(\rho_1 + \rho_2)} = \frac{2 \times 6 \times 3}{9} = 4 \Omega \text{ cm}$$

$$6. \quad V = \frac{4\pi}{3} r^3$$

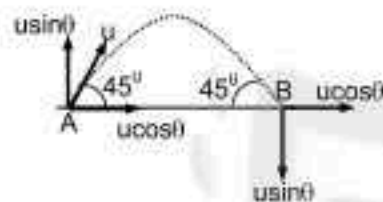
$$\Rightarrow \frac{\Delta V}{V} = 3 \frac{\Delta r}{r} \Rightarrow \text{Relative error}$$

$$\% \text{ error in volume} = \frac{\Delta V}{V} \times 100 = 3 \frac{\Delta r}{r} \times 100 = 3 \times \frac{0.85}{7.50} \times 100 = 33.999 \approx 34$$

7.

$$|\Delta \vec{p}| = 2mu \sin \theta = 2 \times 5 \times 10^{-3} \times 5\sqrt{2} \times \frac{1}{\sqrt{2}} = 5 \times 10^{-2} \text{ kg ms}^{-1}$$

$$\Rightarrow x = 5$$



$$8. \quad \Delta \lambda = \frac{v}{c} \times \lambda = \frac{286}{30000000} \times 630 \times 10^{-9} = 6.006 \times 10^{-10} \approx 6 \times 10^{-10} \text{ m}$$

$$9. \quad \frac{1}{2} kx^2 = \frac{1}{2} mv^2 \Rightarrow x = v \sqrt{\frac{m}{k}} = 10 \times \sqrt{\frac{4}{100}} = 10 \times \frac{2}{10} = 2 \text{ m}$$

$$10. \quad F = \frac{qQ}{4\pi\epsilon_0 r_1^2} + \frac{qQ}{4\pi\epsilon_0 r_2^2} + \frac{qQ}{4\pi\epsilon_0 r_3^2} + \frac{qQ}{4\pi\epsilon_0 r_4^2} + \dots$$

$$\Rightarrow F = \frac{qQ}{4\pi\epsilon_0} \left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \dots \right] \Rightarrow F = \frac{qQ}{4\pi\epsilon_0} \left[\frac{1}{1 - \frac{1}{4}} \right]$$

$$\Rightarrow F = \frac{qQ}{4\pi\epsilon_0} \left[\frac{4}{3} \right] = 10^{-6} \times 1 \times 9 \times 10^9 \times \frac{4}{3} = 12 \times 10^3 \text{ N}$$

PART – B (CHEMISTRY)

SECTION - A

- Bohr's theory accounts for the line spectrum of single electron species but Li^+ has two electrons.
Bohr's theory fails to explain splitting of spectral lines in presence of magnetic field i.e. Zeeman effect.

2. Be is used in X ray tube windows.
Mg is used in incendiary bombs & signals.
Compounds of Ca i.e CaCO_3 is used in extraction of metals like Fe.
Radium (Ra) is used in treatment of cancer in radiotherapy.
3. Both $\text{C}_2\text{H}_5\text{OH}$ and AgCN can generate nucleophile.
 KCN generate nitrile as nucleophile while AgCN generate isonitrile as nucleophile in nucleophilic substitution reaction.
4. Mn_3O_4 is mixed oxide which contains MnO and Mn_2O_3 , so Mn is in +2 and +3 oxidation state respectively. Both Mn^{+2} & Mn^{+3} has unpaired electrons so Mn_3O_4 will show magnetic property. While all other oxides have no unpaired electrons either on cation or on anion.
5. In basic medium H_2O_2 can behaves as both oxidizing and reducing agent, so it can oxidize Mn^{2+} to Mn^{4+} and reduce I_2 to I^- .
6. Covalent solids have high melting point due to strong covalent bonds, also they are insulator in both solid as well as in molten state.
7. Non- biodegradable wastes are generated by thermal power plants which produce fly ash. Biodegradable detergents leads to eutrophication by decreasing oxygen level in water.
8. Hybridisation of carbon a, b, and c respectively are sp^3 , sp^2 and sp^2 .
9. Antifertility drug → Norethindrone
Antibiotic → Salvarsan
Tranquilizer → Meprobamate
Artificial Sweetener → Alitame

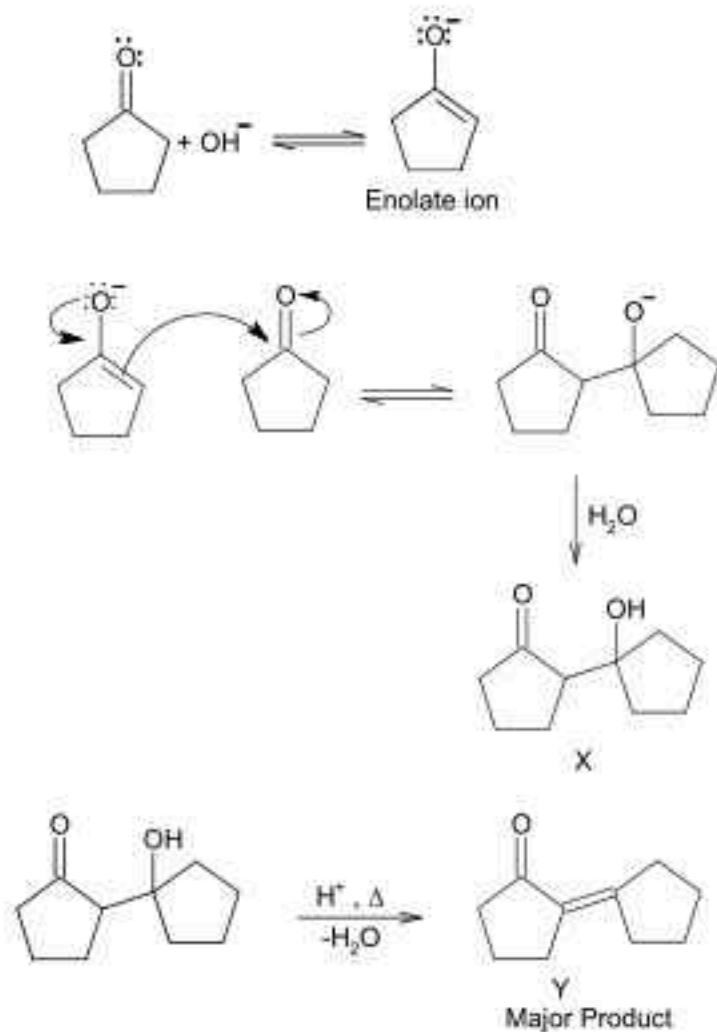
10.

	Oxidation state of N
NO	+2
NO_2	+4
N_2O	+1
NO_3	+5

So, order of oxidation state is
 $\text{NO}_3 > \text{NO}_2 > \text{NO} > \text{N}_2\text{O}$

11. $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$ is blue vitriol which is represented as $[\text{Cu}(\text{H}_2\text{O})_4] \cdot \text{H}_2\text{O} \cdot \text{SO}_4$. Here, secondary valencies are 4 H_2O and 1 H_2O molecule outside coordination sphere is involved in hydrogen bonding.
12. Deficiency of vitamin K causes increase in blood clotting time.
13. In Hoffmann bromamide reaction, hypobromite ion react with amide and in this reaction carbonyl group is lost as CO_2^- in form of Na_2CO_3 .
14. During nitration of aniline, meta- nitroaniline is also formed as product due to formation of $-\text{NH}_3^+$ group. The percentage of p,m and o product is 51%, 47% and 2% respectively.

15.



16. Mercury has low boiling point so is refined by distillation method.

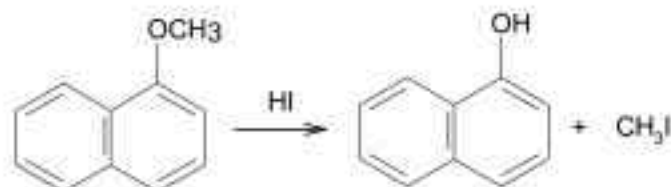
Copper refining is done through electrolytic refining.

Silicon is refined by zone refining method.

Nickel is refined by vapour phase refining.

17. First ionization enthalpy of Mg is smaller than Ar and Cl but higher than Na.

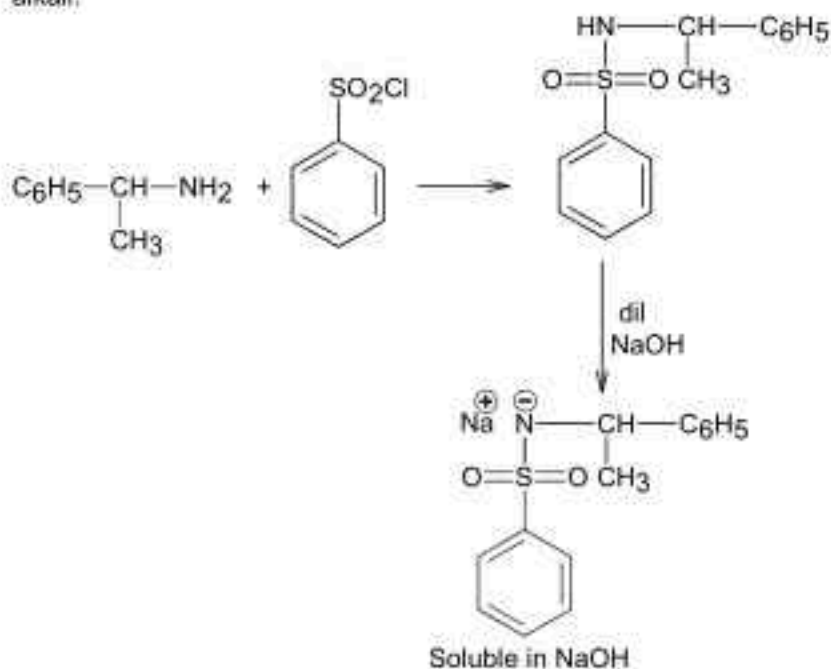
18.



1-methoxy naphthalene

19. Metal sulphide sols are negatively charged while metal oxide sols are positively charged sols. So, CdS is negative while TiO_2 is positively charged sol.

20. Only 1° amine react with benzene sulphonyl chloride to give a compound which is soluble in alkali.



SECTION - B

- $m=2$ molal
 $\Delta T_b = 100.52 - 100 = 0.52^\circ\text{C}$
 Using, $\Delta T_b = iK_b m$
 $0.52 = i \times 0.52 \times 2$
 $i = 0.5$
 Now using,

$$\alpha = \frac{1-i}{1-1/n}$$
 Where, $n=2$ (dimerisation)

$$\alpha = \frac{1-0.5}{1-0.5} = 1$$
 So, percentage association = 100%
- ClF_3 and XeF_4 have two lone pairs of electrons on central atom.
- Partial hydrolysis of XeF_6 gives XeO_2F_2 .
 $\text{XeF}_6 + 2\text{H}_2\text{O} \rightarrow \text{XeO}_2\text{F}_2 + 4\text{HF}$
 Compound A is XeF_6 , so number of lone pair on Xe is 1
- In Tollen's test for aldehyde, aldehyde is oxidized to carboxylic acid salt as:
 $\text{R}-\text{CHO} + \text{H}_2\text{O} \longrightarrow \text{R}-\text{COO}^- + 3\text{H}^+ + 2\text{e}^-$
 So, 2e^- are transferred per aldehyde group.
- $V_{\text{Na}_2\text{CO}_3} = 10$ mL
 Volume of HCl used will be 5 mL (average of titre values)

$$\begin{aligned} \text{Meq of HCl} &= \text{meq of Na}_2\text{CO}_3 \\ (M \times n\text{-factor} \times V)_{\text{HCl}} &= (M \times n\text{-factor} \times V)_{\text{Na}_2\text{CO}_3} \\ 0.2 \times 1 \times 5 &= M \times 2 \times 10 \\ M &= 0.05 \text{ M} \\ M &= 50 \text{ mM} \end{aligned}$$

6. $\lambda_{\text{BaCl}_2}^\circ = 280 \text{ Scm}^2\text{mol}^{-1}$
 $\lambda_{\text{H}_2\text{SO}_4}^\circ = 860 \text{ Scm}^2\text{mol}^{-1}$
 $\lambda_{\text{HCl}}^\circ = 426 \text{ Scm}^2\text{mol}^{-1}$
 $\lambda_{\text{BaSO}_4}^\circ = \lambda_{\text{BaCl}_2}^\circ + \lambda_{\text{H}_2\text{SO}_4}^\circ - 2\lambda_{\text{HCl}}^\circ$
 $= 280 + 860 - 2 \times 426$
 $= 280 \text{ Scm}^2\text{mol}^{-1}$

7. Half life, $t_{1/2} = 1 \text{ min}$
 Let, time of 99.9% completion of reaction be 't' min
 Let the reaction is of first order

$$K = \frac{2.303}{t} \log_{10} \frac{[R]_0}{[R]}$$

$$[R] = 0.001 [R]_0$$

$$\frac{0.693}{1} = \frac{2.303}{t} \log_{10} 10^3$$

$$t = \frac{2.303}{0.693} \times 3 \text{ min}$$

$$t = 9.99 \text{ min}$$

Nearest integer is 10.

8. Moles of benzoic acid = $\frac{6.1}{121}$
 $= 0.05$
 Theoretical moles of m-bromobenzoic acid = 0.05
 Observed moles of m-bromobenzoic acid = $\frac{7.8}{200}$
 $= 0.039$

$$\% \text{ yield} = \frac{0.039}{0.05} \times 100$$

$$= 78\%$$

9. $\Delta G^\circ = +25.2 \text{ kJ/mol}$
 Using $\Delta G^\circ = -2.3 RT \log K_p$
 $25.2 \times 10^3 = -2.3 \times 8.3 \times 400 \log K_p$
 $-3.3 = \log K_p$
 $\log \frac{1}{2 \times 10^3} = \log K_p$
 $K_p = \frac{1}{2 \times 10^3}$
 Using: $K_p = K_c (RT)$
 $\frac{1}{2 \times 10^3} = K_c (0.083 \times 400)^{-1}$

$$K_c = 16.6 \times 10^{-3}$$

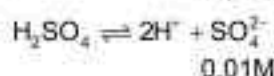
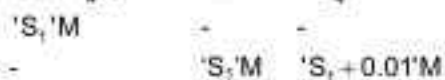
$$K_c = 166 \times 10^{-2}$$

10. Solubility of CdSO_4 in water ; $S = 8 \times 10^{-4} \text{M}$

Using $K_{sp} = S^2$

$$K_{sp} = 64 \times 10^{-8} \text{M}$$

Now,



Using, $K_{sp} = [\text{Cd}^{2+}][\text{SO}_4^{2-}]$

$$64 \times 10^{-8} = S_1(S_1 + 0.01)$$

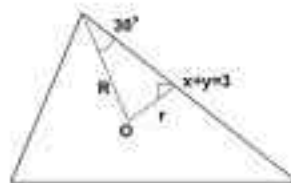
$$S_1 \ll 0.01$$

$$\text{So, } S_1 = 64 \times 10^{-8} \text{M.}$$

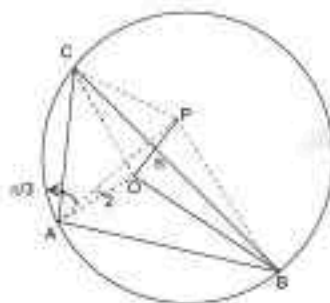
PART – C (MATHEMATICS)

SECTION - A

1. $r = \frac{|0+0-3|}{\sqrt{2}} = \frac{3}{\sqrt{2}}$
 again $\frac{r}{R} = \sin 30^\circ = \frac{1}{2}$
 $R = 2r$
 $\therefore R+r = 3r = \frac{9}{\sqrt{2}}$



2. $\frac{h}{2} = \tan \frac{\pi}{3} = \sqrt{3}$
 $\Rightarrow h = 2\sqrt{3}$



3. $PAP^{-1} = B$ P is non singular
 now $PAP^{-1} = A$ if $P = I$
 \therefore we can say $ARA \Rightarrow$ Reflexive
 $PAP^{-1} = B$
 $\therefore P^{-1}PAP^{-1}P = P^{-1}BP$

$$\Rightarrow A = P^{-1}BP$$

Now PAP^{-1}

$$\Rightarrow PP^{-1}BPP^{-1} = B$$

$$\therefore ARB \Rightarrow BRA$$

$\therefore R$ is symmetric

$$ARB \Rightarrow PAP^{-1} = B \quad \text{--- (I)}$$

$$BRC \Rightarrow PBP^{-1} = C \quad \text{--- (II)}$$

$$\Rightarrow P(PAP^{-1})P^{-1} = C$$

$$\Rightarrow (P^2)A(P^2)^{-1} = C$$

$$\Rightarrow ARC.$$

$\therefore R$ is transitive.

$\therefore R$ is an equivalence Relation.

4. $\frac{x^2}{4} - \frac{y^2}{2} = 1$

$$e = \sqrt{1 + \frac{2}{4}} = \frac{\sqrt{6}}{2}$$

$$F(\sqrt{6}, 0)$$

Equation of tangent at P is $x - \frac{\sqrt{6}y}{2} = 1$

\therefore It cuts x axis at Q(1,0)

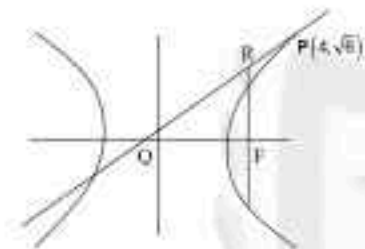
latus rectum at F, $x = \sqrt{6}$ cuts the tangent at.

$$\Rightarrow \sqrt{6} - \frac{\sqrt{6}y}{2} = 1$$

$$y = \frac{2(\sqrt{6}-1)}{\sqrt{6}}$$

$$\text{Area of } \Delta QFR = \frac{1}{2}(\sqrt{6}-1)\left(\frac{\sqrt{6}-1}{\sqrt{6}}\right)$$

$$= \frac{7-2\sqrt{6}}{\sqrt{6}} = \frac{7}{\sqrt{6}} - 2$$



5. $f(x) = \frac{x-2}{x-3}, x \neq 3$ $g(x) = 2x-3$

$$y = \frac{x-2}{x-3} \qquad y = 2x-3$$

$$\Rightarrow x = \frac{3y-2}{y-1} = f^{-1}(y) \quad \Rightarrow x = \frac{y+3}{2} = g^{-1}(y)$$

$$\therefore f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$$

$$\Rightarrow \frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$$

$$\Rightarrow x^2 - 5x + 6 = 0 \Rightarrow x = 2, 3$$

\therefore sum of possible values of x is 5

6. Mean = $\bar{x} = \frac{na - na}{2n} = 0$

$$\text{Variance} = \frac{1}{2n} \sum_{i=1}^{2n} x_i^2 - \bar{x}^2 = \frac{2na^2}{2n} = a^2$$

If b added to all of salvation then

$$\text{mean} = 0 + b = b = 5$$

$$(\text{S.D.})^2 = a^2 = (20)^2$$

$$\therefore a^2 + b^2 = 400 + 25 = 425$$

7. $|\vec{a}| = |\vec{b}| \quad \vec{a} \cdot \vec{b} = 0$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| = |\vec{a}|^2$$

$$\Rightarrow |\vec{b}| = 1 = |\vec{a}|$$

$$\text{Let } \vec{c} = \vec{a} + \vec{b} + \vec{a} \times \vec{b}$$

$$|\vec{c}|^2 = |\vec{a} + \vec{b}|^2 + |\vec{a} \times \vec{b}|^2 + 2(\vec{a} + \vec{b}) \cdot (\vec{a} \times \vec{b})$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{a}|^2$$

$$= 2|\vec{a}|^2 + |\vec{b}|^2 = 3$$

$$\therefore c = \sqrt{3}$$

$$\text{Again } \vec{c} \cdot \vec{a} = |\vec{a}|^2 + (\vec{a} \times \vec{b}) \cdot \vec{a}$$

$$= 1 + 0 = 1$$

\therefore angle between \vec{c} & \vec{a}

$$\cos \theta = \frac{\vec{c} \cdot \vec{a}}{|\vec{c}| |\vec{a}|} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

8. $15 \sin^4 \alpha + 10 \cos^4 \alpha = 6, \alpha \in \mathbb{R}$

$$\Rightarrow 15 \tan^4 \alpha + 10 = 6 \sec^4 \alpha = 6(1 + \tan^2 \alpha)^2$$

$$= 6(1 + 2 \tan^2 \alpha + \tan^4 \alpha)$$

$$\Rightarrow 9 \tan^4 \alpha - 12 \tan^2 \alpha + 4 = 0$$

$$\Rightarrow (3 \tan^2 \alpha - 2)^2 = 0$$

$$\Rightarrow \tan^2 \alpha = \frac{2}{3} \Rightarrow \sin^2 \alpha = \frac{2}{5}, \quad \cos^2 \alpha = \frac{3}{5}$$

$$\therefore 27 \sec^6 \alpha + 8 \cos^6 \alpha = 27 \left(\frac{5}{3}\right)^3 + 8 \left(\frac{5}{2}\right)^3$$

$$= 125 + 125 = 250$$

9.

P	Q	$P \Rightarrow Q$	$\neg Q$	$(P \Rightarrow Q) \wedge \neg Q$	$\neg P$	$((P \Rightarrow Q) \wedge \neg Q) \Rightarrow \neg P$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

10. $\frac{dy}{dx} = (y+1) \left[(y+1)e^{x/2} - x \right]$ - (I)

$$\Rightarrow \frac{1}{(y+1)^2} \frac{dy}{dx} + \frac{x}{(y+1)} = e^{x/2}$$

Let $\frac{1}{y+1} = t \Rightarrow -\frac{1}{(y+1)^2} \frac{dy}{dx} = \frac{dt}{dx}$

$$(I) \Rightarrow \frac{dt}{dx} - xt = -e^{x/2}$$

solution

$$IF, = e^{-\int x dx} = e^{-x/2}$$

$$\frac{e^{-x/2}}{y+1} = -x + c$$

$$y(2) = 0 \Rightarrow e^{-2} = c - 2 \Rightarrow c = 2 + e^{-2}$$

$$\frac{e^{-x/2}}{y+1} = 2 + e^{-2} - x$$

$$x = 1, \Rightarrow y + 1 = \frac{e^{3/2}}{1 + e^2}$$

(I) \Rightarrow

$$\therefore \frac{dy}{dx} = \frac{e^x}{(1+e^2)^2} \times e^{3/2} - \frac{e^{3/2}}{1+e^2} = -\frac{e^{3/2}}{(1+e^2)^2}$$

11. $S_1 = n[a + (2n-1)d]$ - (I)

$$S_2 = 2n[a + (4n-1)d] \dots (ii)$$

$$\begin{aligned}
 S_2 - S_1 &= 2n[a + (4n-1)d] - n[a + (2n-1)d] \\
 &= n[a + (6n-1)d] = 1000 \\
 \therefore S_{6n} &= 3n[a + (6n-1)d] = 3000
 \end{aligned}$$

12. LHL at $x=0$

$$\lim_{x \rightarrow 0} \frac{2 \sin \left\{ \left(\frac{a+3}{2} \right) x \right\} \cos \left\{ \left(\frac{a-1}{2} \right) x \right\} \left(\frac{a+3}{2} \right) x}{\left\{ \left(\frac{a+3}{2} \right) x \right\} \cdot \frac{a+3}{2x}} = b$$

$$\Rightarrow \frac{a+3}{2} = b \Rightarrow a - 2b + 3 = 0 \quad \text{--- (I)}$$

RHL at $x=0$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} = b$$

$$\lim_{x \rightarrow 0} \frac{bx^2}{bx^{3/2} [\sqrt{x+bx^2} + \sqrt{x}]} = b$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+bx^2} + 1} = b$$

$$\Rightarrow b = \frac{1}{2}$$

$$\Rightarrow a = 2b - 3 = -2$$

$$a + b = -2 + \frac{1}{2} = -\frac{3}{2}$$

13. Equation of tangent to $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3} \cos \theta, \sin \theta)$

$$\Rightarrow \frac{x}{3\sqrt{3}} \cos \theta + \frac{y}{1} \sin \theta = 1 \quad \theta \in \left(0, \frac{\pi}{2} \right)$$

\therefore sum of intercepts on coordinate axes

$$z = 3\sqrt{3} \sec \theta + \operatorname{cosec} \theta$$

$$\therefore \frac{dz}{d\theta} = 3\sqrt{3} \sec \theta \tan \theta - \operatorname{cosec} \theta \cot \theta$$

$$\text{for } z \text{ min; } \frac{dz}{d\theta} = 0 \Rightarrow 3\sqrt{3} \sec \theta \tan \theta = \operatorname{cosec} \theta \cot \theta$$

$$\Rightarrow \tan^3 \theta = \frac{1}{3\sqrt{3}} \Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = \frac{\pi}{6}$$

$$\theta > \frac{\pi}{6}, \frac{dz}{d\theta} > 0 \text{ \& } \theta < \frac{\pi}{6}, \frac{dz}{d\theta} < 0$$

$$\therefore z \text{ in minimum for } \theta = \frac{\pi}{6}$$

14. System of equation has not trivial solutions

$$\therefore \Delta = \begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$$R_1 - 2R_2$$

$$\Rightarrow \begin{vmatrix} 0 & \lambda+2 & 0 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow (\lambda+2)(\mu-6) = 0$$

$$\therefore \mu = 6 \text{ \& } \lambda \in \mathbb{R}$$

15. $\frac{1}{3} \leq f(t) \leq 1 \quad \forall t \in [0, 1]$

$$0 \leq f(t) \leq \frac{1}{2} \quad \forall t \in (1, 3]$$

$$g(3) = \int_0^1 f(t) dt + \int_1^3 f(t) dt. \quad - (1)$$

$$\text{Now } \frac{1}{3} \leq \int_0^1 f(t) dt \leq 1 \quad - (2)$$

$$\& 0(3-1) \leq \int_1^3 f(t) dt \leq \frac{1}{2}(3-1)$$

$$\Rightarrow 0 \leq \int_1^3 f(t) dt \leq 1 \quad - (3)$$

$$(2) + (3)$$

$$\Rightarrow \frac{1}{3} \leq \int_0^3 f(t) dt \leq 2$$

$$\Rightarrow g(3) \in \left[\frac{1}{3}, 2 \right]$$

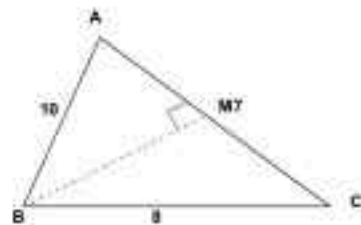
16. $|\overline{BC}| = 8 \quad |\overline{CA}| = 7 \quad |\overline{AB}| = 10$

$a = 8, \quad b = 7, \quad c = 10$

$$\cos A = \frac{49 + 100 - 64}{2(7)(10)} = \frac{17}{28}$$

Projection of AB along AC = $AB \cos A$

$$= 10 \times \frac{17}{28} = \frac{85}{14}$$



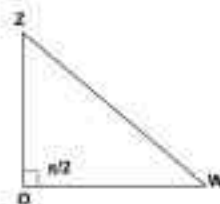
17. $\omega = 1 - \sqrt{3}i$, z is such that $|z\omega| = 1$

$$|\omega| = 2 \quad \Rightarrow |z||\omega| = 1$$

$$\Rightarrow |z| = \frac{1}{2}$$

\therefore Area of Regard triangle

$$= \frac{1}{2} |\omega| |z| = \frac{1}{2}$$



18. $4y^2 = x^2(4-x)(x-2) \geq 0$

$$\Rightarrow x \in [2, 4]$$

$$y = \pm \frac{1}{2} x \sqrt{(4-x)(x-2)}$$

$$A = \frac{1}{2} \int_2^4 x \sqrt{(4-x)(x-2)} dx$$

$$= \frac{1}{2} \int_2^4 x \sqrt{6x - x^2 - 8} dx$$

$$= \frac{1}{2} \int_2^4 (x-3+3) \sqrt{1-(x-3)^2} dx$$

$$= \frac{1}{2} \int_2^4 (x-3) \sqrt{1-(x-3)^2} dx + \frac{3}{2} \int_2^4 \sqrt{1-(x-3)^2} dx$$

put $x-3 = \sin \theta$

$$\Rightarrow dx = \cos \theta d\theta$$

$$A = \frac{1}{2} \int_{-\pi/2}^{\pi/2} \sin \theta \cos^2 \theta d\theta + \frac{3}{2} \int_{-\pi/2}^{\pi/2} \cos^2 \theta d\theta$$

$$= 0 + \frac{3}{2} \int_0^{\pi/2} (1 + \cos 2\theta) d\theta$$

$$A = \frac{3}{2} \left(\theta + \frac{\sin 2\theta}{2} \right) \Big|_0^{\pi/2}$$



$$= \frac{3\pi}{4}$$

$$\therefore \text{Required total area} = 2A = \frac{3\pi}{2}$$

19. p = probability of getting success in a single trial
 q = probability of getting failure in a single trial
 $p+q=1$

$$\text{given } {}^5C_1 p^1 q^4 = 0.4096 \quad \text{--- (I)}$$

$$\& {}^5C_2 p^2 q^3 = 0.2048 \quad \text{--- (II)}$$

$$\frac{{}^5C_1 p q^4}{{}^5C_2 p^2 q^3} = \frac{0.4096}{0.2048} = 2$$

$$\Rightarrow \frac{q}{p} = 4 \Rightarrow p = \frac{1}{5} \quad q = \frac{4}{5}$$

$$\therefore p(x=3) = 5c_3 p^3 q^2 = 10 \times \frac{1}{125} \times \frac{16}{25}$$

$$= \frac{32}{625}$$

20. Let centre of Variable circle is at $C(h,k)$ & radius = r

$$Cc_1 = 3 - r$$

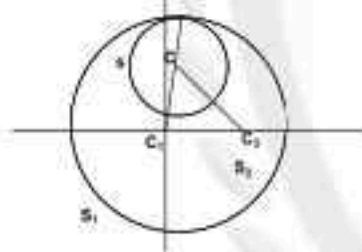
$$Cc_2 = 1 + r$$

$$Cc_1 + Cc_2 = 4 \text{ constant.}$$

\therefore Locus of C is an ellipse.

$$\sqrt{(h-2)^2 + k^2} = 4 - \sqrt{h^2 + k^2}$$

Or, point $\left(2, \pm \frac{3}{2}\right)$ satisfying the expression



SECTION – B

1.
$$\sum_{k=0}^{10} (2^2 + 3k) {}^n C_k = 4 \sum_{k=0}^{10} {}^{10} C_k + 3 \sum_{k=0}^{10} k {}^{10} C_k \quad n = 10 \text{ (assumption)}$$

$$= 4 \cdot 2^{10} + 3 \sum_{k=0}^{10} k \times \frac{10}{k} {}^n C_{k-1}$$

$$= 4 \cdot 2^{10} + 30 \times 2^9$$

$$= 2^{10} (4 + 15) = 19 \times 2^{10} = \alpha \times 3^{10} + \beta \times 2^{10}$$

Comparing $\alpha = 0$ $\beta = 19$

$\therefore \alpha + \beta = 19$

2.
$$P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$$

Its characteristic equation

$$\begin{vmatrix} 2-\lambda & -1 \\ 5 & -3-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 + \lambda - 1 = 0$$

$\therefore P^2 = I - P$ - (i)

$$P^3 = P - P^2 = P - (I - P) = 2P - I$$

$$P^4 = 2P^2 - P = 2(I - P) - P = 2I - 3P$$

$$P^5 = 2P - 3P^2 = 2P - 3(I - P) = 5P - 3I$$

$$P^6 = 5P^2 - 3P = 5(I - P) - 3P = 5I - 8P$$

$\therefore N = 6$

3. $f(x+y) = f(x)f(y)$
 $\Rightarrow f(x) = \lambda^x$
 $f'(x) = \lambda^x \ln \lambda$
 & $f'(0) = \ln \lambda = 3$ & $\lambda = e^3$
 $\therefore f(x) = e^{3x}$
 $\therefore \lim_{x \rightarrow 0} \left(\frac{e^{3x} - 1}{3x} \right) \times 3 = 3$

4. $P(1,3,a)$ **Plane $2x - y + z - b = 0$ ----- - (1)**

$Q(-3,5,2)$

\overline{PQ} is collinear with $2\hat{i} - \hat{j} + \hat{k}$

& $\frac{-4}{2} = \frac{2}{-1} = \frac{2-a}{1} \Rightarrow a = 4$

Mid point of P and Q is $M(-1, 4, 3)$ lies on

$$(l) \Rightarrow -2 - 4 + 3 - b = 0$$

$$\Rightarrow b = -3$$

5. Equation of plane is

$$a(x-1) + b(y+6) + c(z+5) = 0$$

$$\therefore 4a - 3b + 7c = 0 \quad (1)$$

$$\& 3a + 4b + 2c = 0 \quad (2)$$

$$\Rightarrow \frac{a}{-34} = \frac{b}{13} = \frac{c}{25}$$

$$\therefore \text{plane } -34(x-1) + 13(y+6) + 25(z+5) = 0.$$

Now $(1, -1, \alpha)$ lies on it

$$\therefore 65 + 25(\alpha + 5) = 0$$

$$25\alpha = -5(25 + 13)$$

$$5\alpha = -38$$

$$|5\alpha| = 38$$

6. $P'(x) = a(x-1)(x+1) = a(x^2-1)$

$$\therefore P(x) = a\left(\frac{x^3}{3} - x\right) + b$$

$$P(-3) = 0$$

$$\Rightarrow 0 = a(-9+3) + b$$

$$\Rightarrow b = 6a$$

$$\int_{-1}^1 P(x) dx = 18$$

$$\Rightarrow \int_{-1}^1 \left[a\left(\frac{x^3}{3} - x\right) + b \right] dx = 18$$

$$\Rightarrow 2b = 18 \Rightarrow b = 9$$

$$\therefore a = \frac{3}{2}$$

$$\therefore P(x) = \frac{1}{2}x^3 - \frac{3}{2}x + 9$$

\therefore Sum of all coefficients

$$= \frac{1}{2} - \frac{3}{2} + 9 = 8$$

7.
$$\left[\frac{x+1}{x^{\frac{1}{5}} - x^{\frac{2}{5}} + 1} \cdot \frac{x-1}{\sqrt{x}(\sqrt{x}-1)} \right]^{10}$$

$$\Rightarrow \left[(x^{\frac{1}{5}} + 1) \cdot \frac{(\sqrt{x} + 1)}{\sqrt{x}} \right]^{10}$$

$$= \left[x^{\frac{1}{3}} + 1 - x^{-\frac{1}{2}} \right]^{10}$$

$$\text{general term} = {}^{10}C_r x^{\frac{10-r}{3}} \times x^{-\frac{r}{2}} = {}^{10}C_r x^{\frac{20-5r}{6}}$$

$$\text{For independent of } \frac{20-5r}{6} = 0 \Rightarrow r = 4$$

$$\therefore \text{coefficient} = {}^{10}C_4 = 210$$

8.
$$\sum_{r=1}^{10} r! (r^3 + 6r^2 + 2r + 5)$$

$$(r+1)(r+2)(r+3) = r^3 + 6r^2 + 11r + 6$$

$$= \sum_{r=1}^{10} r! [(r+1)(r+2)(r+3) - 9r - 1]$$

$$= \sum_{r=1}^{10} (r+3)! - 9r!(r+1-1) - r!$$

$$= \sum_{r=1}^{10} (r+3)! - 9(r+1)! + 9r! - r!$$

$$= \sum_{r=1}^{10} [(r+3)! - r!] - 9 \sum_{r=1}^{10} [(r+1)! - r!]$$

$$= (11! + 12! + 13! - 1! - 2! - 3!) - 9(11! - 1!)$$

$$= 13! + 12! + 13! - 9! - 9!11! + 9!$$

$$= 13! + 12! + 11! - 9(11!)$$

$$= 13! + 4(11!) = 11!(12 \times 13 + 4) = 160(11!)$$

9.
$$P(x) = f(x^2) + xg(x^2) \quad \text{--- (1)}$$

$x^2 + x + 1 = (x-w)(x-w^2)$ is factor of (1)

$\therefore P(w) = f(1) + wg(1) = 0 \quad \text{--- (2)}$

$P(w^2) = f(1) + w^2g(1) = 0 \quad \text{--- (3)}$

(2) $\times w - (3)$

$f(1)w - f(1) = 0$ & $f(1)(w-1) = 0$

$\Rightarrow f(1) = 0$

Again (2) + (3)

$\& 2f(1) - g(1) = 0 \Rightarrow g(1) = 2f(1)$

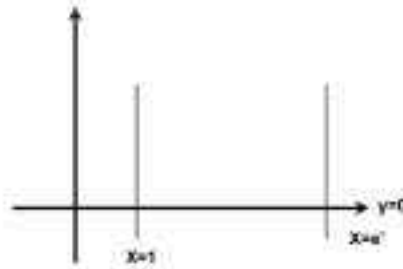
$\therefore P(1) = f(1) + g(1) = 3f(1) = 3 \times 0 = 0$

10.

$$x dy - y dx = \sqrt{x^2 - y^2} \quad x \geq 1.$$

$$d\left(\frac{y}{x}\right) = \frac{1}{x} \sqrt{1 - \left(\frac{y}{x}\right)^2}$$

$$\Rightarrow \int \frac{d(y/x)}{\sqrt{1 - (y/x)^2}} = \int \frac{dx}{x} \Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ln x + c$$



$$\sin^{-1}(0) = 0 + c \Rightarrow c = 0$$

$$y = x \sin(\ln x)$$

$$x = 1, \quad y = 0$$

$$x = e, \quad y = 0$$

$$\text{Required area } A = \int_1^e x \sin(\ln x) dx.$$

$$\text{Let } \ln x = t \Rightarrow \frac{1}{x} dx = dt$$

x	1	e
t	0	π

$$A = \int_0^\pi e^{2t} \sin t dt = \left[\frac{e^{2t}(2 \sin t - \cos t)}{5} \right]_0^\pi = \frac{e^{2\pi}(1+1)}{5} = \frac{2e^{2\pi}}{5} = \alpha e^{2\pi} + \beta$$

$$\therefore \alpha = \frac{2}{5}, \beta = 0$$

$$\therefore 10(\alpha + \beta) = 10\left(\frac{2}{5} + 0\right) = 4$$